SCORE: ______ / 20 POINTS

NO CALCULATORS ALLOWED
SHOW PROPER WORK & SIMPLIFY ALL ANSWERS
(ANSWERS WITHOUT SOLUTIONS WILL *NOT* EARN FULL CREDIT)

Prove the derivative of $\tanh^{-1} x$ without using the logarithmic definition of $\tanh^{-1} x$.
You may use the derivatives of the non-inverse hyperbolic functions that were listed in your textbook without proving them.
NOTE: You must prove the Pythagorean–like identity for $\tanh x$ if you wish to use it.

SCORE: 3 / 4 PTS

$y = \tanh^{-1} x$

$x = \tanh y$ $\frac{1}{2}$

$1 = \frac{dy}{dx} \operatorname{sech}^2 y$ 1

$\frac{dy}{dx} = \frac{1}{\operatorname{sech}^2 y}$ $\frac{1}{2}$ $\frac{1}{1-\tanh^2 y}$

$\therefore \frac{dy}{dx} = \frac{1}{1-x^2}$ $\frac{1}{2}$

* $1 - \tanh^2 x = \operatorname{sech}^2 x$ $\frac{1}{2}$

Find $\frac{d}{dx} \tanh^{-1}(\operatorname{sech} x)$.
You may use the hyperbolic identities and derivatives from your textbook without proving them.

SCORE: 3 / 3 PTS

$\frac{d}{dx} \tanh^{-1}(\operatorname{sech} x) = \frac{1}{1-\operatorname{sech}^2 x} \cdot -(\operatorname{sech} x \cdot \tanh x) = \frac{-\operatorname{sech} x \cdot \tanh x}{\tanh^2 x}$ 1 1

$= \frac{-\operatorname{sech} x}{\tanh x} = -\operatorname{csch} x$

$\frac{1}{2}$ $\frac{1}{2}$

Find $\lim_{x \to 0^-} \operatorname{csch} x$. **Do NOT use a graph. Give *BRIEF* algebraic or numerical reasoning.**

SCORE: ______ / 2 PTS

$\lim_{x \to 0^-} \frac{2}{e^x - e^{-x}} = \frac{2}{1^- - 1^+} = -\infty$ 1

*

Write an algebraic expression to approximate the area under $f(x) = \ln x$ over the interval $[2, 7]$ using a left hand sum and n subintervals (as shown in class). **Do NOT evaluate the expression. Your answer must use $f(\,)$ notation.** **SCORE: ______ / 3 PTS**

$$\Delta x = \frac{7-2}{n} = \frac{5}{n}$$

$$\frac{5}{n}\left\{ f(2) + f\left(2+\frac{5}{n}\right) + f\left(2+\frac{2\cdot 5}{n}\right) \cdots + f\left(2+\frac{(n-1)5}{n}\right)\right\}$$

$$= \frac{5}{n}\left\{ \ln 2 + \ln\left(2+\frac{5}{n}\right) + \ln\left(2+\frac{2\cdot 5}{n}\right) \cdots + \ln\left(2+\frac{(n-1)5}{n}\right)\right\}$$

*

If $\coth x = -3$, find $\operatorname{sech} x$ and $\sinh x$. **SCORE: ______ / 5 PTS**

$\coth^2 x - 1 = \operatorname{csch}^2 x$ *

$(-3)^2 - 1 = \operatorname{csch}^2 x$

$9 - 1 = \operatorname{csch}^2 x$

$\operatorname{csch} x = \pm\sqrt{8}$

$*$ ($\coth x < 0$, $\frac{\cosh x}{\sinh x} = \coth x$; $\cosh x > 0$, so $\frac{1}{\sinh x} < 0$)

$\cosh x = \frac{e^x + e^{-x}}{2}$

$\therefore \operatorname{csch} x = -2\sqrt{2}$

$\therefore \sinh x = -\frac{1}{2\sqrt{2}}$ *

$\cosh^2 x = 1 + \sinh^2 x$

$= 1 + \frac{1}{8}$

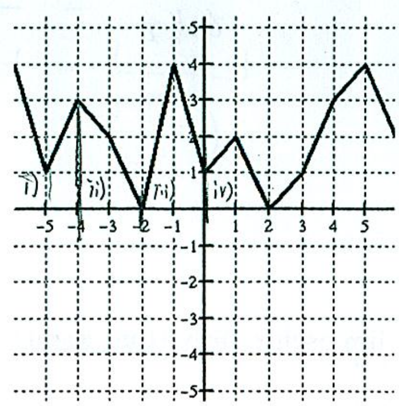
$= \frac{9}{8}$

$\cosh x = \pm\frac{3}{2\sqrt{2}}$

$\therefore \cosh x = \frac{3}{2\sqrt{2}}$ *

$\therefore \operatorname{sech} x = \frac{2\sqrt{2}}{3}$ 1

Estimate the area under the function shown on the right over the interval $[-6, 2]$ using the left hand sum with 4 equal width subintervals. **SCORE: 0 / 3 PTS**



i) $(1+1.5)+(1+1) = 4.5$

ii) $(2+0.5)+1$

iii) $(2)+(1+1.5)$

iv) $(1+0.5)+(1)$